"Basics of "

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With the coming of asymptotic freedom in nonabelian gauge theories in the 1970's it became apparent that partons were nothing other than the quanta which occur in the theory of quark and gluons interacting by means of the gluon couplings to color charge, QCD. Asymptotic freedom allows one to consider quarks as free quanta at short distances and over short times thus making contact with the idea of a parton as a noninteracting constituent of a hadron.

It soon became clear that for many, if not all, high energy processes involving a large momentum transfer one could separate (*factorize*) the process into one part which involves only hard interactions and which is calculable using perturbative QCD and into a second part which requires detailed nonperturbative information as to how hadrons are built out of quarks and gluons. The parts of hard processes involving nonperturbative physics are not energy dependent and can be used in one process after having been measured in another process.

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Our understanding of partons in QCD has progressed rapidly since the 1970's and now QCD partons are used both quantitatively and qualitatively to explain diverse phenomena in both inclusive and exclusive high energy reactions. Sometimes their applications are rather simple conceptually as for example in calculations involving higher order corrections to the electron-positron annihilation cross section into hadrons. Sometimes techniques far beyond traditional renormalization group methods are required as for example in our description of inclusive hadron production in jets. Sometimes it requires great care in deciding what exactly counts as a parton and when a parton counts as a quark and when it counts as a gluon as for example in our discussion of spin-dependent deep inelastic scattering. Sometimes it is not yet clear what are the limits of the usefulness of the parton picture as for example in very small x phenomena.

In this book our approach is completely perturbative. We make no attempt to describe hadron wavefunctions in terms of quarks and gluons. Much of this book is concerned with distributions and correlations of particles produced in, or in association with, jets. However, phenomenologically distributions of hadrons and of partons seem remarkably similar. This has led to hypothesis of local parton hadron duality (LPHD). LPHD has made it possible to apply the predictions of perturbative QCD at much lower energies than would have been the case had the relationship between partons and hadrons not been so close.

If there is a main focus to this book it is obtaining high energy predictions of QCD in circumstances where coherence is important. Little of our discussion is untouched by the idea that gluons couple to quarks and other gluons by means of a conserved color charge.

Some of the predictions discussed in this book are at first sight anti-intuitive as for example the look of a central plateau in jets. Indeed the emergence of the "hump-backed" plateau shape of the inclusive hadron spectrum in jets, recently definitely confirmed at LEP, is perhaps the most striking example of a highly nontrivial prediction of QCD where coherence plays a predominant role.

It is coherence, also, which allows the evolution of a jet from a single parton into a multiparton system to be described, in most of its aspects, in terms of a classical branching process, which description has been so important for understanding the global structure of events in e^+e^- and hadron-hadron collisions.

Color coherence can be used practically as a valuable tool for studying manifestations of new physics. For example, reconstruction of the QCD radiation pattern determinig distributions of accompanying hadrons may help to explore the production of new heavy objects: the Higgs bosons, new quarks, supersymmetric particles, *etc.*

In describing particle distributions and correlations in jets for that region of phase space where most of the particles are produced an unusual circumstance arises. The perturbation theory does not just consist of a few Feynman graphs giving a few powers of α_s, rather both single and double logarithms occur for each power of α_s. This necessitates a resummation of the perturbation series. The summation of all double logarithmic terms, $[\alpha_s \ln^2]^n$, which we refer to as DLA gives a good *qualitative* description of inclusive distributions in jets, however, for a *quantitative* description it is necessary to include single logarithmic terms leading to the modified leading logarithmic approximation, MLLA.

In MLLA distributions are generally down by $\sqrt{\alpha_s}$ compared to DLA. The appearance of $\sqrt{\alpha_s}$ corrections is one of the surprising features of much of the physics we discuss in this book. $\sqrt{\alpha_s}$ terms arise because in the process of summing leading double logs we neglect quantities which have summed up *decreasing* exponentials $\exp(-\sqrt{\alpha_s}\ln)$ while we keep *increasing* exponentials of $\sqrt{\alpha_s}\ln$. Dropping these decreasing exponentials leads to an expansion in $\sqrt{\alpha_s}$ an expansion which has now been reasonably well tested at LEP.

This book is aimed at two rather different audiences. Much of our discussion is descriptive and large parts of the book can be read without going through the details of the derivations of the final results. Thus one of our objectives is to make the book accessible to high energy experimental physicists. We have tried to explain the various phenomena in physical terms and we have stated results often in a directly usable way. On the other hand we have tried to be complete enough in our technical discussions so that the derivations can be followed by an advanced graduate student. Little of what we cover is available in book from elsewhere.

Experimental results are coming in at a very rapid rate, especially from LEP. In order that this book should not age too rapidly we have kept comparisons with experiment to a minimum. We have tried to concentrate on the ideas and on the precise predictions of QCD. Most of the topics we have covered are reasonably mature from a theoretical perspective, though often definitive experimental tests are yet to come.

In the long course of preparing this book we benefited a lot from collaboration with many of our colleagues belonging to the QCD community.

We are especially indebted to B. Andersson, Ya.I. Azimov, S. Bethke, G. Cowan, V.S. Fadin, V.N. Gribov, G. Gustafson, G. Marchesini, P. Mättig, C. Peterson, T. Sjöstrand and B. Webber for fruitful discussions.

This work was completed during the visit of three of us (TDK) at Lund University in the framework of the Nordita Perturbative QCD Workshop. It is a pleasure for us to thank the Theory Group of Lund University for having arranged a nice time for physics there.

Chapter 1

Perturbative Approach to Hard Processes and Jets

Perturbative (hereafter — PT) QCD aims to describe quantitatively the structure of multipartonic systems produced by QCD cascades for gaining some actual knowledge about confinement from comparing the calculable characteristics of quark-gluon ensembles with measurable characteristics of final hadronic states in hard processes.

It has become a matter of folklore to say that QCD has taken over from the old Parton Model. This is however only a half truth. Indeed, if one is thinking of the Hard Processes such as $e^+e^- \to$ ***hadrons*** or deeply inelastic lepton-hadron scattering (DIS) this is just the case: it was QCD that supplied the heuristic parton picture with the dynamical basis. At the same time it would be wasteful to forget that the main advantage of the Parton Model [1] was just a universal approach to both "hard" and "soft" physics. The parton idea has explained the phenomenon of scaling in structure functions (hard DIS and e^+e^- processes) and in the inclusive particle distributions in hadron collisions at the same time. It was the parton picture which predicted a jet-like structure of final states of hard interactions and, on the other hand, which gave a clear explanation of Regge asymptotics [2].

Modern QCD is far from reaching such a universality, the impassable *infrared gulf* still lies between soft and hard processes. A characteristic of the QCD burst of the last 15 years — certain coolness to a class of problems of Reggistics, does not mean a nonurgency of the topic. Simply it remains to be too hard a nut to crack for the modern theory.

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Meanwhile the need of an integrated approach to the description of both hard and soft hadroproduction processes becomes imminent. It is the experimentally observed universality of quark distributions and jets, similar particle content in the hard lepton-hadron and soft hadron-hadron collisions *etc.* which point to this need.

To elaborate such an integrated approach one has to gain better understanding of the confinement mechanism, to search for more detailed qualitative and quantitative information about the region of really strong interaction where the quark-gluon language becomes inapplicable and starting from the QCD field Lagrangian we are unable to keep track of the happenings. To enter the problem we should start with portraying the space-time picture of the hadroproduction in hard processes *i.e.* drawing the picture of "blanching" of colored partons — quarks and gluons.

1.1 Space-Time Picture of QCD Bremsstrahlung

Now we proceed to the consideration of the basic hard interaction, namely the creation and propagation of a quark q and an antiquark $\overline{q}$ in the annihilation process $e^+e^- \to q\overline{q}$.

The key problem here is how to organize a color neutral final system of "white" hadrons from the initial pair of colored quarks flying apart with relativistic velocities from the annihilation point. At first sight it seems to be rather difficult (if not even impossible) to reconcile the fast spreading of color with the final color neutrality needed. How to let quarks know that they should not take away color (and fractional electric charges by the way)?! Here we'll concentrate on this puzzle which furnishes the very core of the hadronization problem.

Firstly let us remind the reader a standard answer to the above question: "*there exists the non-perturbative "gluon string" or the "flux tube" of strong chromomagnetic field between quarks (color charges) which starts to stretch and breaks producing new $q\overline{q}$ pairs to form hadrons subsequently.* This incantation however has little to do with the problem under interest. Why so? Let us catch sight of this point.

The concept of the *string* (the "area law" in the Wilson loop, the linearly rising potential and all that) could in principal explain "*why* do quarks form a white hadron" but not "*how* does this confinement occur in the fast non-adiabatic process". The reason for this is that relativistic quantum mechanics (together with asymptotic freedom) protects the fast quark from being involved in any non-PT interaction during a long time. The original q and $\bar{q}$ come out of the PT jurisdiction and can enter the "hadronization game" only after macroscopically large time interval from the start of the process, which is proportional to the quark energy: $t_{hadr} \propto E$.

1.1.1 Field Regeneration Time and Hadronization

The essence of a "hard process" is that the quark is knocked out from the vacuum (or from a hadron as in DIS process) as a *bare* particle or, more accurately, as a *half-dressed* one. This means that the charge, when being accelerated, appears to have a truncated proper field (either electro- or chromo-magnetic). Its field has no Fourier components with $k_\perp < \sqrt{Q^2}$ where Q^2 denotes the characteristic momentum transfer squared measuring the "hardness" of the hard process.

Subsequently two closely correlated processes start: the bremsstrahlung is developing whose quanta in time leave the parent radiating quark as offspring partons — gluons, and the private gluonic field of the quark is regenerated. The phenomenon of regeneration of a stationary field surrounding a charge has been understood in QED. The regeneration time of the proper field or, to be more precise, of its Fourier component with momentum $\vec{k}$ is given by

$$T_{regen.}(k) \sim \frac{k_{\|}}{k_\perp^2}\,, \tag{1.1}$$

where longitudinal and transverse components of photon momentum are defined with respect to the outgoing electron. For photons with relatively small transverse momenta $k_\perp \ll k_{\|} \sim p \approx E$ where p and E stand for electron momentum and energy, the regeneration time may become macroscopically large. The finiteness of the regeneration time leads to substantial effects for relativistic electrons.

When the fast electron is scattered at large angle, two cones of bremsstrahlung photon radiation are well known to be formed (see Fig. 1.1a). The quanta from the first cone centered around the direction of an initial electron momentum can be treated as splinters of an electromagnetic surrounding shaken off by the accelerated electron. An appearance of the second cone is the back of the regeneration of a new field coat *i.e.* the disk of the Lorentz contracted Coulomb field fitting the final electron.

Next, for a double scattering process depicted in Fig. 1.1b it might seem natural to expect an appearance of the *four* photon cones. However

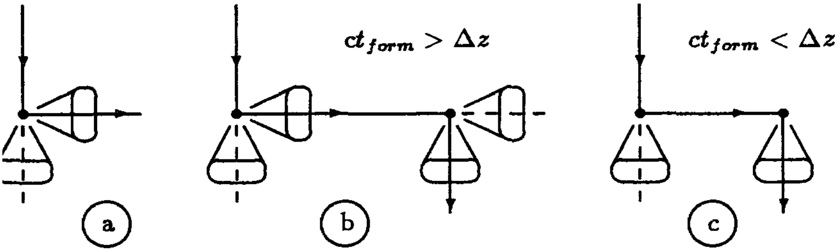

Figure 1.1: Electromagnetic bremsstrahlung accompanying electron scattering.

if the regeneration length $cT_{regen.}$ (1.1) exceeds the distance between the scattering points, only two of them actually emerge (Fig. 1.1c).

The regeneration physics shows up in a number of QED phenomena connected with relativistic electrons, such as the Landau-Pomeranchuk damping of soft bremsstrahlung due to multiple rescatterings of electron in a medium, in transmission radiation, electron radiation in crystals *etc.*

Classical considerations alone give evidence of a truncated field. Let us take the classical charge which is moving along z axis with velocity $v \approx 1$ after being accelerated (say, from a $v = 0$ state) at $t = 0$. At asymptotically large time it will be surrounded by a disk of Lorentz contracted e.m. field. It is clear however that such a state could not emerge instantly. In the reference frame accompanying the charge the field spreads out inside the sphere

$$r' \leq t'.$$

This means that in the laboratory frame where the time is slowered by the factor $\gamma = E/m$, the field at distances r from apart the z axis will appear not earlier then at

$$t = \gamma t' = \frac{E}{m} r. \tag{1.2}$$

Applied to a quark this results in a rather serious consequence: an energetic "bare" quark prepared by a hard interaction will be able to hadronize (*i.e.* to become a hadron constituent) only after the time (1.2), where r measures typical value of interquark distances inside a hadron (hadronic size R) and m should be treated as its constituent mass. For a light quark ($q = u, d, s$) these two parameters are closely linked to each other and to the value of mean transverse momenta characteristic for soft hadron physics:

$$m_{constituent} \sim \sqrt{\langle k_\perp^2 \rangle} \sim R^{-1} \approx \text{few hundred MeV} \tag{1.3}$$

Thus for light and heavy ($Q = c, b, \ldots$) quarks one arrives at the following estimates of ***hadronization time***:

$$t_q^{hadr} \approx ER^2 \,, \tag{1.4}$$

$$t_Q^{hadr} \approx \frac{E}{m_Q} R \,. \tag{1.5}$$

The same conclusions could be drawn from

Quantum-Mechanical arguments. Indeed, in the rest frame of a hadron the confining forces stem from "long-wave" gluonic field with momentum components

$$k'_\perp \sim k'_{||} \sim k' \sim R^{-1} \,.$$

Moving back to the Lab. system one gets

$$k_\perp = k'_\perp \sim R^{-1}, \quad k_{||} = \gamma k'_{||} = \frac{E}{mR} \,,$$

and the time (1.2) one needs for such a field to be regenerated reads:

$$T_{regen.}(k) = \frac{k_{||}}{k_\perp^2} \approx \frac{E}{mR} R^2 = \frac{E}{m} R \,,$$

which coincides with the classical formula Eq.(1.5).

Thus a quark with energy $E \sim 200\,\text{GeV}$ starting from the annihilation time $t^{ann.} \sim 1/E \sim 10^{-3}\,\text{fm}/c$ and up to hadronization time $t^{hadr} \sim ER^2 \sim 10^3\,\text{fm}/c$ should behave as a true color particle radiating gluons perturbatively without any care taken about its future confinement. An instructive lesson comes from considering an *ultraheavy* quark Q heavier than the mass of the weak boson W: $m_Q > 100\,\text{GeV}$. Due to the semiweak decay ($Q \to W + q$) its lifetime τ_Q

$$\tau_Q \approx 1\,\text{fm}/c \left(\frac{M_W}{m_Q}\right)^3 \frac{E_Q}{m_Q} \quad < \quad t_{hadr} \approx 1\,\text{fm}/c \, \frac{E_Q}{m_Q}$$

is shorter than the hadronization time so that for all its life it remains under the jurisdiction of PT QCD. One can say that such a quark in all aspects behaves as if it were a free colored object.